OPE analysis for polarized deep inelastic scattering

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Abstract

We present an explicit OPE analysis for the first moment of g_1 up to order M^2/Q^2. This result allows to calculate power corrections to the Bjorken and Ellis–Jaffe sum rules.

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The investigation of the spin structure of the nucleon is one of the most active fields in hadron physics. This is demonstrated by the fact that CERN, SLAC, and DESY all have programs in polarized lepton–hadron scattering. So far data are available from EMC (proton) [1], SMC (deuteron) [2], and E142 (helium) [3]. As all of these groups took data at different values of Q^2 these results can not be compared directly. In particular for E142 the lowest x points are measured at only $Q^2 \approx 1\ \mathrm{GeV}^2$ such that higher twist corrections should be important. Actually one has to face a typical track–off. At low Q^2 one can get very good statistics but the theoretical analysis is more involved, at high Q^2 theory is easy, but the statistics of the experiment does not allow detailed statements.

Polarized scattering has, however, the great advantage that the higher twist corrections seem to be calculable with a sufficient precission [4]. As also the HERMES experiment at HERA will run at small Q^2 it is a necessary task for theory to clarify the issue of Q^2 dependence. The most basic block for this is the operator product expansion for the moments of g_1 and g_2. Unfortunately there was a mistake in the original work [5] as noticed by Ji and Unrau [6]. The whole discussion was plagued by the fact that often the relevant definitions and conventions are not stated explicitely. Unluckily the real or apparent differences left often the impression that the higher twist corrections are kind of undefined. This is not true. OPE gives a unique result and by now all groups working on this problem have reached the same results. In this paper we present our calculation in detail with all definitions given, such that no misunderstandings should remain.

As indicated the aim of this paper is to identify higher twist operators, which contribute at the next–to leading twist level. We do not however attempt to calculate $O(\alpha_s)$ corrections to the Wilson coefficients.

The starting point of the calculation is the forward virtual Compton scattering amplitude:

$$T_{\mu\nu}(q,P,S) = \mathrm{i} \int d^4\xi \; e^{\mathrm{i}q\cdot\xi} \langle PS| T\left(j_\mu(\xi)\; j_\nu(0)\right) |PS\rangle \; . \tag{1}$$

The target nucleon has four momentum P^μ ($P^2 = M^2$) and spin S^μ ($S^2 = -M^2$, $S \cdot P = 0$). The virtual photon has four momentum q^μ , ($q^2 = -Q^2 \leq 0$). The nucleon state is normalized covariantly $\langle PS|P'S'\rangle = 2P^0(2\pi^2)\delta^3(\vec{P} - \vec{P}')\delta_{SS'}$. Using the analyticity properties of $T_{\mu\nu}$

$$\frac{1}{2\mathrm{i}}\left[T_{\mu\nu}(q_0 + \mathrm{i}\varepsilon) - T_{\mu\nu}(q_0 - \mathrm{i}\varepsilon)\right] = -2\pi W_{\mu\nu} \tag{2}$$

and equation (1) we find the following form for the hadronic tensor:

$$W_{\mu\nu} = \frac{1}{4\pi} \int d^4\xi \; e^{\mathrm{i}q\cdot\xi} \langle PS| \left[j_\mu(\xi), j_\nu(0)\right]_- |PS\rangle \; . \tag{3}$$

We define commutators and anticommutators as $\{A, B\}_+ = AB + BA$ and $[A, B]_- = AB - BA$. The antisymmetric part of the hadronic tensor can be decomposed as follows:

$$W^A_{\mu\nu} = \mathrm{i}\varepsilon_{\mu\nu\sigma\lambda}\frac{q^\lambda}{\nu}\left((g_1(x,Q^2) + g_2(x,Q^2))S^\sigma - g_2(x,Q^2)P^\sigma \frac{q\cdot S}{\nu}\right) \; , \tag{4}$$

with $\nu = p \cdot q$, the Bjorken variable $x = Q^2/2\nu$ and $\varepsilon^{0123} = 1$.

We are only interested in g_1 and g_2, so we consider only the antisymmetric part of $T_{\mu\nu}$.

Performing a Cauchy integration one obtains the connection between $T^A_{\mu\nu}$ and $W^A_{\mu\nu}$, which scematicaly reads [8]:

$$\int_0^1 W^A_{\mu\nu} x^n \; dx = \frac{1}{8\pi\mathrm{i}} \oint d\omega \; T^A_{\mu\nu}(\omega)\omega^{-n-2} \; , \tag{5}$$

with $\omega = 1/x$. Equation (5) holds for $n = 0$ only if $T^A_{\mu\nu}(\omega)$ vanishes fast enough for $\omega \to \infty$. General considerations based on Regge theory suggest that the part of $T^A_{\mu\nu}$ that determines g_1 goes rapidly enough to zero, but the part, which determines g_2 may not [8]. In this publication we assume the validity of equation (5) for all $n \in \mathbf{N}_0$.

Starting from equation (1) one obtains a representation of $T^A_{\mu\nu}$ as a series in ω in the kinematical domain where $|\,\omega\,| \leq 1$, i.e.:

$$T^A_{\mu\nu} = \sum_m b^{(m)}_{\mu\nu}\omega^m \; . \tag{6}$$

Where from equation (5) it follows that:

$$\int_0^1 W^A_{\mu\nu} x^n \; dx = \frac{1}{4} b^{(n+1)}_{\mu\nu} \; . \tag{7}$$

The only Feynman amplitudes which contribute to $W^A_{\mu\nu}$ up to $1/Q^2$ order are shown in Figure 1 [5]. To proceed further we follow the method proposed by Shuryak and Vainsthein [5]. In their formalism contribution of Figure 1 to equation (1) has the form:

$$T^A_{\mu\nu}(q,P,S) = -A_{(\mu\nu)} \sum_f \int d^4\xi \, \langle PS|(\xi|\bar{\psi}_f q_f^2 \left(\gamma_\mu \frac{1}{\not{P}+\not{q}}\gamma_\nu + \gamma_\nu \frac{1}{\not{P}-\not{q}}\gamma_\mu\right)\psi_f|0)|PS\rangle \ . \tag{8}$$

The symbol $A_{(\mu\nu)}$ denotes antisymmetrization of the indices μ and ν. Through this paper we use the Schwinger notation [7], in which $|\xi)$ denotes a formal eigenvektor of the coordinate operator X_μ, i.e.:

$$X_\mu|\xi) = \xi_\mu|\xi) \ , \tag{9}$$

while P_μ is the momentum operator which satisfies

$$\begin{aligned} [P_\mu, X_\nu]_- &= \mathrm{i} g_{\mu\nu} \ , \\ [P_\mu, P_\nu]_- &= \mathrm{i} g G_{\mu\nu} \ . \end{aligned} \tag{10}$$

In the coordinate basis P_μ acts as the covariante derivative

$$(\xi|P_\mu|\xi') = \mathrm{i}\frac{\partial}{\partial\xi^\mu}\delta(\xi-\xi') + gA_\mu(\xi)\delta(\xi-\xi') \ . \tag{11}$$

In this representation the propagator for massless quarks can be written as

$$S(\xi,0) = (\xi|\frac{1}{\not{P}}|0) \ , \ \ S(0,\xi) = -(0|\frac{1}{\not{P}}|\xi) \ . \tag{12}$$

In addition to derive equation (8) we have used the following identities:

$$e^{-\mathrm{i}q\cdot\xi}P_\mu = (P_\mu + q_\mu)e^{-\mathrm{i}q\cdot\xi} \ , \ \ e^{-\mathrm{i}q\cdot\xi}|0) = |0) \ . \tag{13}$$

To avoid misunderstanding in the following we will denote by round brackets, e.g. $(0|\ldots|\xi)$, the matrix elements in the formal coordinate space, to be distinguished from square brackets denoting matrix elements between physical hadronic states, e.g. $\langle PS|\ldots|PS\rangle$. The Dirac spinor of the quark of flavor f is denoted ψ_f and q_f is its charge. The sum runs over the light flavors (u,d,s) in the nucleon under consideration. Note that P denotes the impuls *operator* in these formulas, while q and ξ are c–numbers. The matrix element between proton states in (8) is taken at the scale Q^2. To evaluate equation (8) we expand $1/(\not{P}+\not{q})$ and $1/(\not{P}-\not{q})$:

$$\begin{aligned} \frac{1}{\not{P}+\not{q}} &= \frac{1}{\not{q}} - \frac{1}{\not{q}}\not{P}\frac{1}{\not{q}} + \frac{1}{\not{q}}\not{P}\frac{1}{\not{q}}\not{P}\frac{1}{\not{q}} - \frac{1}{\not{q}}\not{P}\frac{1}{\not{q}}\not{P}\frac{1}{\not{q}}\not{P}\frac{1}{\not{q}} + \ldots \ , \\ \frac{1}{\not{P}-\not{q}} &= -\frac{1}{\not{q}} - \frac{1}{\not{q}}\not{P}\frac{1}{\not{q}} - \frac{1}{\not{q}}\not{P}\frac{1}{\not{q}}\not{P}\frac{1}{\not{q}} - \frac{1}{\not{q}}\not{P}\frac{1}{\not{q}}\not{P}\frac{1}{\not{q}}\not{P}\frac{1}{\not{q}} + \ldots \ . \end{aligned} \tag{14}$$

The l.h.s. is some non–local operator, while the r.h.s. gives local differential operators by identifying $P_\mu = \mathrm{i}D_\mu = \mathrm{i}\partial_\mu + gA_\mu$, with the covariant derivative D_μ (see Eq. (11)). Inserting only the first term of the expansion into equation (8) yields $T^A_{\mu\nu}$ in the lowest order.

$$
\begin{aligned}
T^A_{\mu\nu} &= -\frac{1}{q^2}\sum_f \int d^4\xi \langle PS|(\xi|\bar\psi_f q_f^2 \left(\gamma_\mu \not{q} \gamma_\nu - \gamma_\nu \not{q} \gamma_\mu\right)\psi_f|0)|PS\rangle \\
&= 2\mathrm{i}\varepsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda}{q^2}\sum_f \langle PS|\bar\psi_f(0) q_f^2 \gamma^\sigma\gamma^5\psi_f(0)|PS\rangle \\
&= \mathrm{i}\varepsilon_{\mu\nu\sigma\lambda}\frac{q^\lambda S^\sigma}{\nu}\,\omega\,(2\sum_f q_f^2 a_f^{(0)})\ ,
\end{aligned}
\tag{15}
$$

where we have used the identity

$$
\gamma_\mu\gamma_\lambda\gamma_\nu - \gamma_\nu\gamma_\lambda\gamma_\mu = 2\mathrm{i}\varepsilon_{\mu\nu\lambda\sigma}\gamma^5\gamma^\sigma\ , \tag{16}
$$

($\gamma^5 = \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3$) and have introduced the matrix element of the local operator

$$
\langle PS|\bar\psi_f(0)\gamma^\sigma\gamma^5\psi_f(0)|PS\rangle = 2S^\sigma a_f^{(0)}\ . \tag{17}
$$

For $T^A_{\mu\nu}$ in the lowest order this gives:

$$
\begin{aligned}
\int_0^1 \left(g_1(x) + g_2(x)\right)\ dx &= \frac{1}{2}a^{(0)} \\
\int_0^1 g_2(x)\ dx &= 0\ ,
\end{aligned}
\tag{18}
$$

with $a^{(0)} = \sum_f q_f^2 a_f^{(0)}$. This is the well known result:

$$
\int_0^1 g_1(x)\ dx = \frac{1}{2}a^{(0)}\ . \tag{19}
$$

The corrections to this sum rule are obtained by calculating the next term contributing to $T^A_{\mu\nu}$ denoted by $\delta T^A_{\mu\nu}$.

$$
\delta T^A_{\mu\nu} = -\frac{1}{q^6}\sum_f \int d^4\xi \langle PS|(\xi|\bar\psi_f q_f^2 \left(\gamma_\mu \not{q}\not{P}\not{q}\not{P}\not{q}\gamma_\nu - \gamma_\nu \not{q}\not{P}\not{q}\not{P}\not{q}\gamma_\mu\right)\psi_f|0)|PS\rangle\ . \tag{20}
$$

Using the equations of motion we get[1]:

$$
\gamma_\mu \not{q}\not{P}\not{q}\not{P}\not{q}\gamma_\nu - \gamma_\nu \not{q}\not{P}\not{q}\not{P}\not{q}\gamma_\mu = 4\varepsilon_{\mu\nu\lambda\delta}\varepsilon^{\delta}{}_{\alpha\beta\sigma}q^\lambda q^2\ \ P^\alpha P^\beta\gamma^\sigma - 8\mathrm{i}\varepsilon_{\mu\nu\lambda\sigma}q^\lambda q_\alpha q_\beta\ \ P^\alpha P^\beta\gamma^\sigma\gamma^5\ . \tag{21}
$$

[1] due to the EOM $\int d^4\xi \big(\xi\big|\bar\psi_f \ldots \not{P}\psi_f\big|0\big) = 0 = \int d^4\xi\big(\xi\big|\bar\psi_f\not{P}\ldots\psi_f\big|0\big)$

In the above formula we suppressed $\int d^4\xi(\xi|\bar{\psi}_f \dots \psi_f|0)$. Now we decompose the Lorentz tensor of rank three $P^\alpha P^\beta \gamma^\sigma \gamma^5$ into parts with spin three to zero:

$$\begin{aligned}
&P^\alpha P^\beta \gamma^\sigma \gamma^5 = \\
&O_{5\ \text{spin 3}}^{\alpha\beta\sigma} + \tilde{O}_{5\ \text{spin 2}}^{\alpha\beta\sigma} + \frac{5}{18} g^{\alpha\beta} P^2 \gamma^\sigma \gamma^5 - \frac{1}{18} g^{\alpha\sigma} P^2 \gamma^\beta \gamma^5 - \frac{1}{18} g^{\beta\sigma} P^2 \gamma^\alpha \gamma^5 \\
&+ \frac{\mathrm{i}}{6} \varepsilon^{\alpha\beta\sigma\lambda} g \tilde{G}_{\lambda\delta} \gamma^\delta \gamma^5 \ ,
\end{aligned} \tag{22}$$

and equivalent for $P^\alpha P^\beta \gamma^\sigma$. $\tilde{G}_{\lambda\delta}$ is the dual QCD field tensor

$$\tilde{G}_{\lambda\delta} = \frac{1}{2} \varepsilon_{\lambda\delta\alpha\beta} G^{\alpha\beta} \ . \tag{23}$$

The operators O are defined as follows:

$$\begin{aligned}
O_{5\ \text{spin 3}}^{\alpha\beta\sigma} = \frac{1}{6} \Big(& P^\alpha P^\beta \gamma^\sigma + P^\alpha P^\sigma \gamma^\beta + P^\beta P^\alpha \gamma^\sigma \\
&+ P^\sigma P^\alpha \gamma^\beta + P^\beta P^\sigma \gamma^\alpha + P^\sigma P^\beta \gamma^\alpha \Big) \gamma^5 \\
&- \frac{1}{18} \left(P^2 g^{\alpha\beta} \gamma^\sigma + P^2 g^{\beta\sigma} \gamma^\alpha + P^2 g^{\alpha\sigma} \gamma^\beta \right) \gamma^5 \ ,
\end{aligned} \tag{24}$$

$$\begin{aligned}
\tilde{O}_{5\ \text{spin 2}}^{\alpha\beta\sigma} \ = \ & \frac{1}{3} \left(2P^\alpha P^\beta \gamma^\sigma - P^\sigma P^\alpha \gamma^\beta - P^\beta P^\sigma \gamma^\alpha \right) \gamma^5 \\
& - \frac{1}{9} \left(2P^2 g^{\alpha\beta} \gamma^\sigma - P^2 g^{\beta\sigma} \gamma^\alpha - P^2 g^{\alpha\sigma} \gamma^\beta \right) \gamma^5 \ .
\end{aligned} \tag{25}$$

The operators $O_{\text{spin 3}}^{\alpha\beta\sigma}$ and $\tilde{O}_{\text{spin 2}}^{\alpha\beta\sigma}$ are the same as $O_{5\ \text{spin 3}}^{\alpha\beta\sigma}$ and $\tilde{O}_{5\ \text{spin 2}}^{\alpha\beta\sigma}$, they just go without the γ^5 matrix. The spin one part of equation (22) is checked to be the correct one by taking suitable traces, the spin zero part by contraction with $\varepsilon_{\alpha\beta\sigma\lambda'}$. The spin three and two operators are traceless. The spin three part is known to have the form of equation (24) [9]. $\tilde{O}_{5\ \text{spin 2}}^{\alpha\beta\sigma}$ is now determined unique, because it is the only operator that fulfills the identity (22). Although this spin two operator has no specific symmetry properties, it can be written as a linear combination of two tensors with mixed symmetry. This corresponds to the fact, that a tensor of rank three contains two irreducible representations with spin two (as three vectors can be coupled to spin two in two different ways).

Inserting equations (21) and (22) into equation (20) gives:

$$\begin{aligned}
&\delta T_{\mu\nu}^A = \\
&= \sum_f \Big[-\langle PS|\bar{\psi}_f(0) q_f^2 \Big(O_{\text{spin 3}}^{\alpha\beta\sigma} + \tilde{O}_{\text{spin 2}}^{\alpha\beta\sigma} + \frac{5}{18} g^{\alpha\beta} P^2 \gamma^\sigma - \frac{1}{18} g^{\alpha\sigma} P^2 \gamma^\beta \\
&- \frac{1}{18} g^{\beta\sigma} P^2 \gamma^\alpha + \frac{\mathrm{i}}{6} \varepsilon^{\alpha\beta\sigma\rho} g \tilde{G}_{\rho\delta} \gamma^\delta \Big) \psi_f(0)|PS\rangle \cdot 4\varepsilon_{\mu\nu\lambda\delta'} \varepsilon^{\delta'}{}_{\alpha\beta\sigma} q^\lambda \frac{1}{q^4} \\
&+ \langle PS|\bar{\psi}_f(0) q_f^2 \Big(O_{5\ \text{spin 3}}^{\alpha\beta\sigma} + \tilde{O}_{5\ \text{spin 2}}^{\alpha\beta\sigma} + \frac{5}{18} g^{\alpha\beta} P^2 \gamma^\sigma \gamma^5 - \frac{1}{18} g^{\alpha\sigma} P^2 \gamma^\beta \gamma^5 \\
&- \frac{1}{18} g^{\beta\sigma} P^2 \gamma^\alpha \gamma^5 + \frac{\mathrm{i}}{6} \varepsilon^{\alpha\beta\sigma\lambda} g \tilde{G}_{\lambda\delta} \gamma^\delta \gamma^5 \Big) \psi_f(0)|PS\rangle \cdot 8\mathrm{i}\varepsilon_{\mu\nu\lambda\sigma} q^\lambda q_\alpha q_\beta \frac{1}{q^6} \Big] \ .
\end{aligned} \tag{26}$$

For symmetry reasons only the following terms contribute to $\delta T^A_{\mu\nu}$

$$
\begin{aligned}
&\delta T^A_{\mu\nu} = \\
&= \sum_f \Big[-4\langle PS|\bar\psi_f(0) q_f^2 \frac{\mathrm{i}}{6}\varepsilon^{\alpha\beta\sigma\rho} g\tilde G_{\rho\delta}\gamma^\delta \psi_f(0)|PS\rangle \cdot \varepsilon_{\mu\nu\lambda\delta'}\varepsilon^{\delta'}{}_{\alpha\beta\sigma} q^\lambda \frac{1}{q^4} \\
&+\langle PS|\bar\psi_f(0) q_f^2 \Big(O^{\alpha\beta\sigma}_{5\ \mathrm{spin}\ 3} + O^{\alpha\beta\sigma}_{5\ \mathrm{spin}\ 2} + \frac{5}{18} g^{\alpha\beta} P^2\gamma^\sigma\gamma^5 - \frac{1}{18} g^{\alpha\sigma}P^2\gamma^\beta\gamma^5 \\
&-\frac{1}{18} g^{\beta\sigma}P^2\gamma^\alpha\gamma^5 \Big) \psi_f(0)|PS\rangle \cdot 8\mathrm{i}\varepsilon_{\mu\nu\lambda\sigma} q^\lambda q_\alpha q_\beta \frac{1}{q^6}\Big] \ . \qquad (27)
\end{aligned}
$$

Note that due to the contraction only O_5, but not O contribute to g_1 and g_2. Additional only that part of $\tilde O^{\alpha\beta\sigma}_{5\ \mathrm{spin}\ 2}$ that is symmetric in $\alpha\beta$ ($O^{\alpha\beta\sigma}_{5\ \mathrm{spin}\ 2}$) contributes. With

$$
\begin{aligned}
\tilde O^{\alpha\beta\sigma}_{5\ \mathrm{spin}\ 2} &= \frac{1}{2}\Big((\mathbf{1}+\mathbf{P}_{\alpha\beta})\tilde O^{\alpha\beta\sigma}_{5\ \mathrm{spin}\ 2} + (\mathbf{1}-\mathbf{P}_{\alpha\beta})\tilde O^{\alpha\beta\sigma}_{5\ \mathrm{spin}\ 2}\Big) \\
&= O^{\alpha\beta\sigma}_{5\ \mathrm{spin}\ 2} + \frac{1}{2}\Big((\mathbf{1}-\mathbf{P}_{\alpha\beta})\tilde O^{\alpha\beta\sigma}_{5\ \mathrm{spin}\ 2}\Big) \ , \qquad (28)
\end{aligned}
$$

we get

$$
\begin{aligned}
&O^{\alpha\beta\sigma}_{5\ \mathrm{spin}\ 2} = \\
&\frac{1}{6}\Big(2P^\alpha P^\beta\gamma^\sigma + 2P^\beta P^\alpha\gamma^\sigma - P^\sigma P^\alpha\gamma^\beta - P^\sigma P^\beta\gamma^\alpha - P^\beta P^\sigma\gamma^\alpha - P^\alpha P^\sigma\gamma^\beta\Big)\gamma^5 \\
&-\frac{1}{9}\Big(2P^2 g^{\alpha\beta}\gamma^\sigma - P^2 g^{\beta\sigma}\gamma^\alpha - P^2 g^{\alpha\sigma}\gamma^\beta\Big)\gamma^5 \ . \qquad (29)
\end{aligned}
$$

$\mathbf{P}_{\alpha\beta}$ is the operator, which changes the indices α and β. As stressed by Shuryak and Vainshtein [5], every step of this calculation is gauge invariant ($0 = q^\mu T_{\mu\nu} = q^\nu T_{\mu\nu}$).

The matrix elements of the relevant local operators are as follows:

$$
\begin{aligned}
&\langle PS|\bar\psi_f(0) O^{\alpha\beta\sigma}_{5\ \mathrm{spin}\ 3}\psi_f(0)|PS\rangle = \\
&= 2a_f^{(2)}\Big[\frac{1}{6}\Big(P^\alpha P^\beta S^\sigma + P^\beta P^\alpha S^\sigma + P^\sigma P^\alpha S^\beta + P^\beta P^\sigma S^\alpha + P^\alpha P^\sigma S^\beta + P^\sigma P^\beta S^\alpha\Big) \\
&-\frac{M^2}{18}\Big(g^{\alpha\beta}S^\sigma + g^{\beta\sigma}S^\alpha + g^{\alpha\sigma}S^\beta\Big)\Big] \ , \qquad (30)
\end{aligned}
$$

$$
\begin{aligned}
&\langle PS|\bar\psi_f(0) O^{\alpha\beta\sigma}_{5\ \mathrm{spin}\ 2}\psi_f(0)|PS\rangle = \\
&= \frac{1}{6}\langle PS|\bar\psi_f(0)\Big[\gamma^\alpha g\tilde G^{\beta\sigma} + \gamma^\beta g\tilde G^{\alpha\sigma}\Big]\psi_f(0)|PS\rangle - \mathrm{traces} = \\
&= 2d_f^{(2)}\Big[\frac{1}{6}\Big(2P^\alpha P^\beta S^\sigma + 2P^\beta P^\alpha S^\sigma - P^\beta P^\sigma S^\alpha - P^\alpha P^\sigma S^\beta - P^\sigma P^\alpha S^\beta - P^\sigma P^\beta S^\alpha\Big) \\
&-\frac{M^2}{9}\Big(2g^{\alpha\beta}S^\sigma - g^{\beta\sigma}S^\alpha - g^{\alpha\sigma}S^\beta\Big)\Big] \ , \qquad (31)
\end{aligned}
$$

$$\langle PS|\bar{\psi}_f(0)P^2\gamma_\alpha\gamma_5\psi_f(0)|PS\rangle = \langle PS|\bar{\psi}_f(0)g\tilde{G}_{\alpha\beta}\gamma^\beta\psi_f(0)|PS\rangle = 2M^2 f_f^{(2)} S_\alpha \,. \qquad (32)$$

With this definitions [2] for the matrix elements and equation (7) one gets from $T^A_{\mu\nu}$ and $\delta T^A_{\mu\nu}$:

$$\begin{aligned}
\int_0^1 dx\ \Big(g_1(x,Q^2)+g_2(x,Q^2)\Big) &= \frac{1}{2}a^{(0)} + \frac{M^2}{9Q^2}\Big(a^{(2)}+4d^{(2)}+4f^{(2)}\Big)\ , \\
\int_0^1 dx\ g_2(x,Q^2) &= 0\ , \\
\int_0^1 dx\ \Big(g_1(x,Q^2)+g_2(x,Q^2)\Big)\,x^2 &= \frac{1}{6}a^{(2)} + \frac{1}{3}d^{(2)}\ , \\
\int_0^1 dx\ g_2(x,Q^2)x^2 &= -\frac{1}{3}a^{(2)} + \frac{1}{3}d^{(2)}\ ,
\end{aligned} \qquad (33)$$

with

$$a^{(n)} = \sum_f q_f^2 a_f^{(n)}\ ,\quad d^{(n)} = \sum_f q_f^2 d_f^{(n)}\ ,\quad f^{(n)} = \sum_f q_f^2 f_f^{(n)}\ . \qquad (34)$$

Including the next term of the expansion in $T_{\mu\nu}$ would result in corrections of the order $(Q^2/M^2)^2$ for the first moments, corrections of the order Q^2/M^2 for the third moments, and it would give values for the fifth moments. We can therefore conclude:

$$\int_0^1 dx\ g_1(x,Q^2) = \frac{1}{2}a^{(0)} + \frac{M^2}{9Q^2}\Big(a^{(2)}+4d^{(2)}+4f^{(2)}\Big) + O(\frac{M^4}{Q^4})\ , \qquad (35)$$

$$\int_0^1 dx\ g_2(x,Q^2) = 0 + O(\frac{M^4}{Q^4})\ , \qquad (36)$$

$$\int_0^1 dx\ g_1(x,Q^2)x^2 = \frac{1}{2}a^{(2)} + O(\frac{M^2}{Q^2})\ , \qquad (37)$$

$$\int_0^1 dx\ g_2(x,Q^2)x^2 = -\frac{1}{3}a^{(2)} + \frac{1}{3}d^{(2)} + O(\frac{M^2}{Q^2})\ . \qquad (38)$$

Equation (36) states that up to twist four and in the leading order of α_s there are no terms which contribute to the first moment of g_2, i.e. which would violate the Burkhardt–Cottingham sum rule [11]. The sign in front of $f^{(2)}$ in equation (35) is positive. This is in agreement with the result of Shuryak and Vainshtein [5] and Balitsky, Braun, Kolesnichenko [4]. In their revised paper [6] Ji and Unrau have derived the $f^{(2)}$ term with a negative sign, but this is only due to their convention for D_μ, see footnote to equation (32). We want to stress that all three calculations agree with each other when the differences due to different conventions are taken into account.

With $a^{(2)}_{\rm proton} = 0.022 \pm 0.002$ from the E130 [12] and the EMC [1] measurement, $a^{(0)}_{\rm NS}$ and $a^{(0)}_{\rm S}$ from the EMC measurement, and the estimates $d^{(2)}_{\rm proton} = -1.4\cdot 10^{-3} \pm 4.0\cdot 10^{-3}$

[2] Note the sign difference in equation (32) with respect to Ji and Unrau [6], owing to the fact, that they use $D_\mu = \partial_\mu + \mathrm{i}gA_\mu$.

($d_N^{(2)} = -0.026 \pm 0.004$) and $f^{(2)}_{\rm proton} = -0.050 \pm 0.018$ ($f_N^{(2)} = 0.018 \pm 0.018$), which we get from the QCD sum rule calculation of Balitsky, Braun and Kolesnichenko [4] one obtaines

$$\int_0^1 g_1^p(x,Q^2)\,dx = \tag{39}$$
$$= 0.124 \cdot (1 - \frac{\alpha_s(Q^2)}{\pi}) + 0.013 \cdot (1 - \frac{33-8n_f}{33-2n_f}\frac{\alpha_s(Q^2)}{\pi}) - (0.018 \pm 0.0072) \cdot \frac{\rm GeV^2}{Q^2}\ .$$

The QCD radiative corrections are taken from [13]. Figure 2 shows this dependence of the first moment of g_1 on Q^2 for low Q^2 ($n_f = 3$). Our result agrees with the result of ref. [4]. Note, however, that in their publications $\gamma^5_{BBK} = -\gamma^5$ and therefore $S_{\sigma\ BBK} = \bar N \gamma_\sigma \gamma^5_{BBK} N = -2S_\sigma$. The different sign in front of the Q^2 corrections of equation (39) compared to the sign of Ji and Unrau [6] (equation (29)) is due to fact, that although their calculation agrees with ours, numerically the bag model prediction for the matrix element $f^{(2)}_{\rm proton}$ has opposite sign than the predictions from QCD sum rules.

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Figure Caption

Figure 1: Graphs contributing to $T\,(j_\mu(\xi)\;j_\nu(0))$.

Figure 2: Dependence of the first moment of g_1^P on Q^2 (solid line). The dotted line shows only the radiative corrections. The dashed line corresponds to $-0.018/Q^2$.